**H. Algebra 2 NOTES: Quadratic Functions: DAY 2**

**Transformations of Quadratic Functions: Transforming Investigation(handout)**

- The parent quadratic function is____________
- Vertex form for a quadratic function is:_______________

Graph the parent quadratic function. Then graph each of the following quadratic functions and describe the transformation.

| Function | Transformation |
|---|---|
| $f(x)=x^2+4$ | |
| $f(x)=x^2-6$ | |
| $f(x)=(x+2)^2$ | |
| $f(x)=(x-5)^2$ | |
| $f(x)=-x^2$ | |
| $f(x)=3x^2$ | |
| $f(x)=\frac{1}{4}x^2$ | |
| $f(x)=2(x-6)^2+7$ | |
| $f(x)=-3(x+2)^2-5$ | |

SUMMARY:

- What causes a reflection?
- What causes a translation to the right/left?
- What causes a translation up/down?
- What causes a vertical compression?
- What causes a vertical stretch?

Minimum/maximum Value

- If a > 0, the parabola opens upward. The **y-coordinate** of the vertex is the___________________.
- If a < 0, the parabola opens downward. The **y-coordinate** of the vertex is the __________________.

PRACTICE

Given a function, name the vertex, the axis of symmetry, the maximum or minimum value, the domain, and the range.

1) $y=3(x-4)^2-2$

2) $y=-2(x+1)^2+3$

Describe the transformation from $y = x^2$ to each of the following:

1) $y = (x-6)^2 + 7$ 2) $y = .2(x-12)^2 - 3$ 3) $y = -.25x^2 + 3$

Day 3: Standard Form to Vertex Form, Vertex Form to Standard Form, and Applications of Quadratic Functions.

## Summary of Quadratic Functions(Day 2)

The equation $y = a(x - h)^2 + k$ is in VERTEX FORM, where (h, k) is the _______.

h controls the ________ and k controls the _________.

1. Give the vertex and describe the graph of $y = 3(x - 2)^2 + k$ as compared to the parent graph of $y = x^2$.

2. Give the vertex and describe the graph of $y = \frac{-1}{2}(x+4)^2 - 8$ as compared to the parent graph of $y = x^2$.

SUMMARY:

- The standard form of the quadratic equation is_____________________.
- The vertex form of the quadratic equation is ______________________.

Write an equation in **vertex form** given the following:

1) Vertex (0, 0) Point(2, 1) 2) Vertex (1, 2) Point (2, -5) 3) Vertex (-3, 6) Point (1, -2)

4) Vertex (-1, -4) , Point: y-intercept is 3 5) Vertex (3, 6) , Point: y-intercept is 2

6) Vertex (0, 5) Point (1, -2)

Change from standard form to vertex form:

1) $y = x^2 - 4x + 5$ 2) $y = x^2 + 6x + 11$

**Application of Quadratic Functions**

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**Example 1:** A model for a company's revenue is $\mathbf{R = -15p^2 + 300p + 12{,}000}$, where **p** is the price in dollars of the company's product. What price will maximize his revenue? Find the maximum revenue.

**Example 2**: Using your calculator, find a quadratic function that includes the following points: (1, 0), (2, -3), and (3, -10).

**Your try:**

A. Find a quadratic function with a graph that includes (2, -15), (3, -36), and (4, -63).

B. An object is dropped from a height of 1700 ft above the ground. The function $h = -16t^2 + 1700$ gives the object's height **h** in feet during free fall at **t** seconds.
   a. When will the object be 1000 ft above the ground?

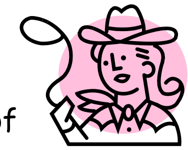
   b. When will the object be 940 ft above the ground?

   c. What are a reasonable domain and range for the function h?

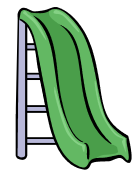
C. When serving in tennis, a player tosses the tennis ball vertically in the air. The height,h, of the ball after t seconds is given by the quadratic function: $h(t) = -5t^2 + 7t$ (the height is measured in meters from the point of the toss).
   a. How high in the air does the ball go?

   b. Assume that the player hits the ball on its way down when it's 0.6 m above the point of the toss. For how many seconds is the ball in the air between the toss and the serve?

**Example 3:** A rancher is constructing a cattle pen by the river. She has a total of 150 feet of fence and plans to build the pen in the shape of a rectangle. Since the river is very dee, she need only fence 3 sides of the pen. Find the dimensions of the pen so that it encloses the maximum area



**Example 4:** A town is planning a child care facility. The town wants to fence in a playground area using one of the walls of the building. What is the largest playground area that can be fenced in using 100 feet of fencing?



# Quadratic Regression!!(Day 3)

*Quadratic Regression is a process by which the equation of a parabola of "best fit" is found for a set of data.*

1. Write the equation of the parabola that passes through the points (0, 0), (2, 6), (-2,6), (1, 1), and (-1, 1).

2. Use the data in the table to find a model for the average weekly sales for the Flubo Toy Company. Do you think a linear model would work? How about a quadratic model?

    - Find the Regression Equation
    - Describe the correlation
    - Predict the sales for the $9^{th}$ week.

| Week | Sales (millions) |
|---|---|
| 1 | $15 |
| 2 | $24 |
| 3 | $29 |
| 4 | $31 |
| 5 | $30 |
| 6 | $25 |
| 7 | $16 |
| 8 | $5 |

3. The concentration (in milligrams per liter) of a medication in a patient's blood as time

passes is given by the data in the following table:

a) What is the quadratic equation that models this situation?

b) What is the concentration of medicine in the blood after 2.25 hours have passed?

| Time (Hours) | Concentration (mg/l) |
|---|---|
| 0 | 0 |
| 0.5 | 78.1 |
| 1 | 99.8 |
| 1.5 | 84.4 |
| 2 | 50.1 |
| 2.5 | 15.6 |

4. A ball is tossed from the top of a building. This table shows the

height, h (in feet), of the ball above the ground t seconds after being tossed.

| t(time) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| h(height) | 299 | 311 | 291 | 239 | 155 | 39 |

- Find the quadratic best-fit model.
- According to a quadratic best-fit model of the data, how long after the ball was tossed was it 80 feet above the ground?
- According to the quadratic best-fit model, what height will the ball reach in 3.7 seconds?

5. The table below gives the stopping distance for an automobile under certain road conditions.

| Speed(mi/h) | 20 | 30 | 40 | 50 | 55 |
|---|---|---|---|---|---|
| Stopping distance (ft) | 17 | 38 | 67 | 105 | 127 |

a. Find a linear model for the data.

b. Find a quadratic model for the data.

c. Compare the models. Which is better? Why?

# Factoring Polynomials—Day 4

- Factors
  - Recall: When 2 or more numbers are multiplied to form a product, each number is a "factor" of the product.
    - Factors of 12: ________________________

- Factoring Polynomials

  - ALWAYS factor out the ________________ ______________ ______________ (_______) FIRST!!!
  - A polynomial that can not be factored is ____________.
  - A polynomial is considered to be completely factored when it is expressed as the product of prime polynomials.

---

A. Factoring out the GCF:

a. $16m^2n + 12mn^2$   b. $14a^3b^3c - 21a^2b^4c + 7a^2b^3c$   c. $7xy + 2ab$

B. Factor by grouping—for polynomials with 4 or more terms

a. $a^2x + b^2x + a^2y + b^2y$   b. $3x^3 + 2xy - 15x^2 - 10y$

c. $20ab - 35b - 63 + 36a$

C. Factoring trinomials into the product of two binomials

a. When leading coefficient is one.

i. $x^2 + 5x + 4$   ii. $x^2 + 6x - 16$

iii. $x^2 - 2x - 63$   iv. $a^2 - 9a + 20$

v. $x^2 + 5x + 6$

b. When leading coefficient is not one("Bustin up the B")

i. $3x^2 - 6x - 24$ ii. $4x^2 + 7x + 3$ iii. $6n^2 + 25n + 14$

iv. $20a^2 - 21a - 5$ v. $20a^2 - 21a - 5$

D. Difference of "Two Squares"

a. Rule: ________________________________________

i. $x^2 - 25$ ii. $16x^4 - z^4$ iii. $6x^2 - 600y^2$

**Concept Summary: Polynomial Factoring Techniques**

| Techniques | Examples |
|---|---|
| **1. Factoring out the GCF**<br><br>Factor out the greatest common factor of all the terms | $15x^4 - 20x^3 + 35x^2$ |
| **2. Factoring by Grouping**<br><br>$ax + ay + bx + by$<br>$= a(x+y) + b(x+y)$<br>$= (a+b)(x+y)$ | $x^3 + 2x^2 - 3x - 6$ |
| **3. Quadratic Trinomials**<br><br>"Bustin up the B" | $6x^2 + 11x - 10$ |
| **4. Difference of Two Squares**<br><br>$a^2 - b^2 = (a-b)(a+b)$ | $25x^2 - 49$ |

**Let's Practice our Factoring:**

1) $2x^2 + 9x + 10$ 2) $-4x^2 + 2x + 30$ 3) $-7x^2 + 175$

4) $-10x^2 + x + 21$ 5) $2x^2 - 3x - 15$ 6) $5x^2 - 20$

**7) The area in square meters of a rectangular parking lot is $x^2 - 95x + 2100$. The width is x – 60. What is the length of the parking lot in meters?**

# Day 5: Solve Quadratic Equations by Factoring and Graphing

**5.5: Solving Quadratic Equations by Factoring**

**To solve a quadratic equation by factoring, you must remember your**

**___________________________________!**

**Zero Product Property:**

- Let A and B be real numbers or algebraic expressions. If AB=0, then A=0 or B=0.
- This means that If the product of 2 factors is zero, then at least one of the 2 factors had to be zero itself!
-

Example: Solve. A) $x^2 + 3x - 18 = 0$ B) $2t^2 - 17t + 45 = 3t - 5$ C) $3x - 6 = x^2 - 10$

**Finding the Zeros of an Equation**

- The zeros of an equation are the ____________!
- First, change y to a _________.
- Now, solve for ____.
- The solutions will be the ________ of the equation.

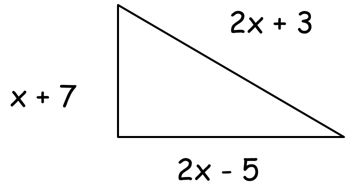
Example: Find the zeros of
$y = x^2 - x - 6$

**You TRY!!!**

1. $2x^2 - 11x = -15$ 2. $16x^2 = 8x$

3. The height of a rectangular prism is 2 feet. The length of the prism is 3 feet more than its width. The volume of the prism is 20 cubic feet. Find the dimensions of the prism.

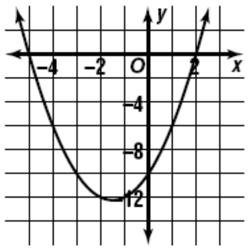
4. Find the numeric dimensions of the right triangle with a hypotenuse of (2x + 3)J meters, and legs of (2x – 5) and (x + 7) meters.



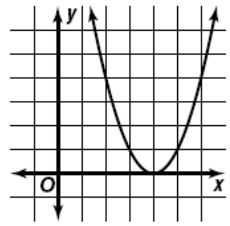
**Solve by Graphing: DAY 6**

**State the real roots of each quadratic equation whose related function is graphed below.**

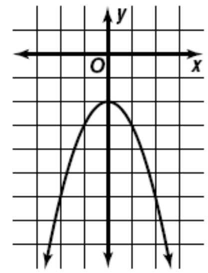
**1.**



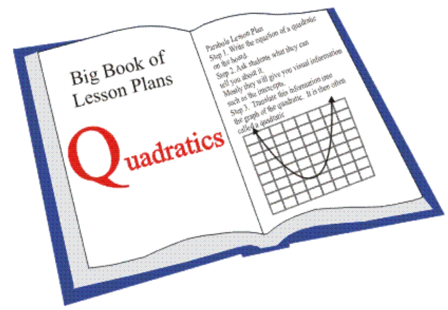

**2.**




**3.**




**Example:** **Solve $x^2 - 5x + 2 = 0$**




**1. Graph in your calculator.**

**2. Use the CALC feature to find the two zeros of the function.**

You try:

**a.** $x^2 + 6x + 4 = 0$ **b.** $3x^2 + 5x - 12 = 8$

## Applications: Solve by graphing

1. A woman drops a front door key to her husband from their apartment window several stories above the ground. The function $y = -16t^2 + 64$ gives the height **h** of the key in feet, **t** seconds after she releases it.

a. How long does it take the key to reach the ground?

b. What are the reasonable domain and range for the function **h**?

2. You use a rectangular piece of cardboard measuring 20 in. by 30 in. to construct a box. You cut squares with sides x in. from each corner of the piece of cardboard and then fold up the sides to form the bottom.

a. Write a function A representing the area of the base of the box in terms of x.

b. What is a reasonable domain for the function A?

c. Write an equation if the area of the base must be 416 in.$^2$.

d. Solve the equation in part© for values of x in the reasonable domain.

e. What are the dimensions of the base of the box?

## Day 8: Simplify Radicals

To simplify a radical, factor the expression under the radical sign to its prime factors. For every pair of like factors, bring out one of the factors. Multiply whatever is outside the sign, then multiply whatever is inside the sign. Remember that for each pair, you "bring out" only one of the numbers.

***Examples*:** $\sqrt{28}$
7 4
7 2 2
$2\sqrt{7}$

$\sqrt{54}$
9 6
3 3 3 2
$3\sqrt{2 \times 3} = 3\sqrt{6}$

$\sqrt{150}$
15 10
3 5 2 5
$5\sqrt{3 \times 2} = 5\sqrt{6}$

$\sqrt{720}$
72 10
9 8 2 5
3 3 2 2 2 2 5
$3 \times 2 \times 2\sqrt{5} = 12\sqrt{5}$

Simplify completely:

1. $\sqrt{9} =$
2. $\sqrt{32}$
3. $\sqrt{50}$
4. $\sqrt{80}$
5. $\sqrt{72}$
6. $\sqrt{120}$
7. $\sqrt{68}$
8. $\sqrt{200}$
9. $\sqrt{180}$
10. $\sqrt{33}$
11. $3\sqrt{12}$
12. $5\sqrt{48}$
13. $2\sqrt{76}$
14. $-3\sqrt{32}$
15. $5\sqrt{80}$

## Honors Algebra 2 ~ Unit 3 ~ Day 7 5.6 Complex Numbers with Conjugates

### 1. Pure Imaginary Numbers

Solve: $3x^2 + 15 = 0$

*There is* ____________________ *that when squared results in a negative number.*

About 400 years ago, Rene Descartes proposed that the solution to the equation $x^2 = -1$ be represented by a number *i*, where *i* is not a real number. This number was given the name "imaginary number" because people could not believe that such numbers existed.

The _____________________________, **i**, can be used to describe the square roots of negative numbers.

$$i = \sqrt{-1}$$

When there is a negative radicand under a square root, you must take out ***i*** before simplifying or performing operations!!

Simplify.

| | | |
|---|---|---|
| **1.** $\sqrt{-9}$ | **2.** $\sqrt{-12}$ | **3.** $\sqrt{-72}$ |
| **4.** $3i \bullet 8i$ | **5.** $\sqrt{-6} \bullet \sqrt{-10}$ | **6.** $(3\sqrt{-5})^2$ |

- **The trick is to remember that the powers of *i* work in cycles of four:**

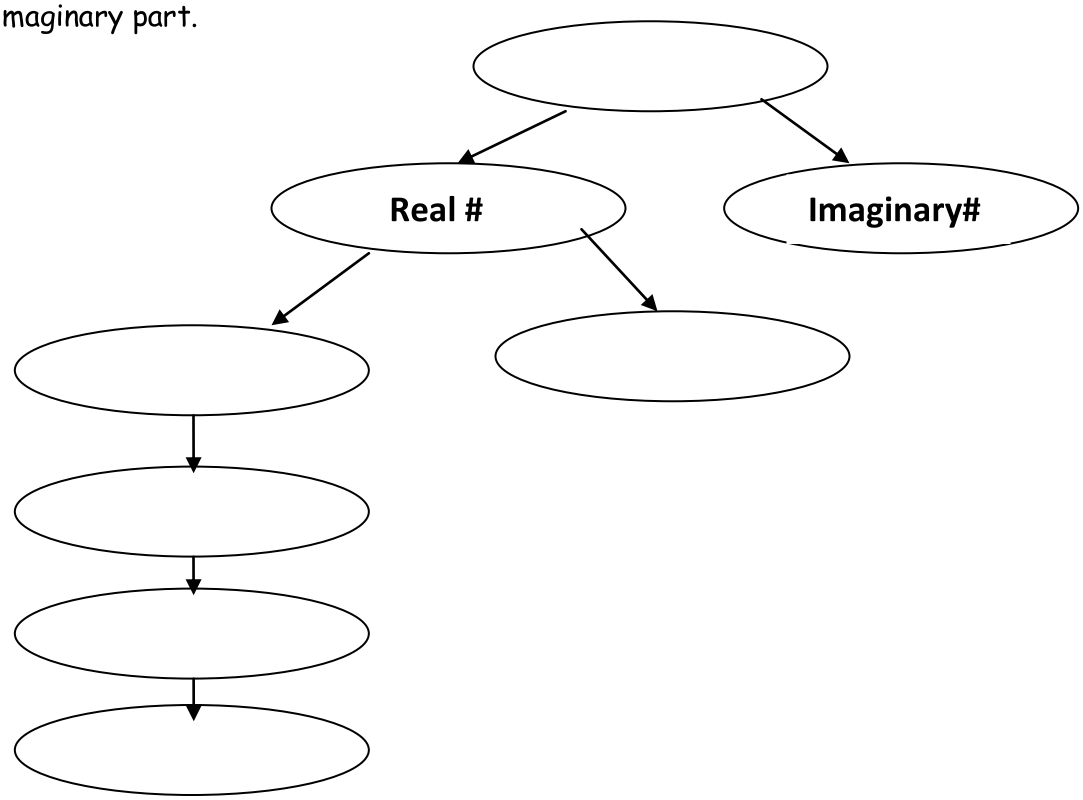
- $i^1 = i$
- $i^2 = \sqrt{-1} \times \sqrt{-1} = (\sqrt{-1})^2 = -1$
- $i^3 = \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} = \sqrt{-1} \times (\sqrt{-1})^2 = -i$
- $i^4 = \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} = (\sqrt{-1})^4 = 1$

## A. Equations with Imaginary Solutions:

**Solve by taking the square root of each side**:

$3x^2 + 48 = 0$ $\qquad$ $a^2 + 72 = 0$

A _____________ is any number that can be written in the form a + b*i* where **a** and **b** are real numbers and ***i*** is the imaginary unit. a is called the real part, and b is called the imaginary part.



*Simplify*

$8-5i \;+\; 2+i$ $\qquad$ $4+7i \;-\; 2+3i$ $\qquad$ $4+2i \;\; 3-5i$ $\qquad$ $9+7i \;\; (9-7i)$

**When dividing, an imaginary number cannot be left in the denominator**(remember **i** is the square root of -1 and we have a rule to not leave any roots in a denominator). **To eliminate the imaginary number in the denominator, multiply by the complex conjugate.**

**The complex conjugate of a + bi is:______________________________.**

$\frac{5}{1+i}$ $\qquad$ $\frac{5+2i}{3-2i}$ $\qquad$ $\frac{2+i}{3i}$

## Equate Complex Numbers

Find the values of x and y that make the equation

$2x-3+\;\; y-4\;\; i=3+2i$ true. $\qquad$ $m+1\;\; +3ni=5-9i$

**Ways to Solve a Quadratic Equation**

Factoring

Graphing

Taking the Square root

Completing the Square

Quadratic Formula

**Solve by "Taking the square root"**

Steps

1. Isolate "$x^2$" or the squared term by using inverse operations.

2. Square root both sides to isolate "x".

3. Give the positive and negative answer.

Examples:

#1 $x^2 = 49$ #2 $a^2 + 72 = 0$ #3 $3x^2 + 15 = 0$

#4 $(x+7)^2 = 36$ #5 $3x^2 + 40 = -x^2 - 56$

## Day 10: **Solve Quadratic Equations by Completing the Square**

**(Demonstration using Algebra Tiles)**

**http://www.regentsprep.org/Regents/math/algtrig/ATE12/completesq.htm**

- **Step 1**: MOVE THE CONSTANT TO THE OTHER SIDE!
-
- **Step 2: A** MUST BE EQUAL TO 1! IF IT IS NOT 1, FACTOR OUT **A**
-
- **Step 3**: FIND HALF OF **B**!
-
- **Step 4**: SQUARE THE # ABOVE AND ADD TO BOTH SIDES OF THE EQUATION!
- (If you factored out A you must multiply the # by A before you add it to the other side!!...you will see this soon!!)

- **Step 5**: REWRITE THE TRINOMIAL AS A BINOMIAL SQUARED!
- Remember a binomial squared looks like $(x + \_\_\_\_)^2$.
- The number in the blank will always be ______________.
-
- **STEP 6:** Find the square root of each side and solve for x.

Examples:

1) $x^2 - 12x + 5 = 0$ 2) $x^2 - 8x + 36 = 0$ 3) $2x^2 = 6x - 20$

4) $4x^2 + 4x - 11 = 0$ 5) $2x^2 + 8x - 10 = 0$ 6) $3x^2 - 12x + 7 = 0$

HONORS ALGEBRA 2 Unit 4~ Day 11

**Quadratic Formula and the Discriminant**

| |
|---|
| Quadratic Equation:<br><br>Quadratic Formula: |

For any quadratic equation in the form $ax^2 + bx + c = 0$, the quantity $\sqrt{b^2 - 4ac}$ is

called the ______________________________.

| Quadratic Equation | Discriminant | Nature of the Roots | Related Graph and Number of x-intercepts |
|---|---|---|---|
| $3x^2 + 2x - 8 = 0$ | | | |
| $-3x^2 + 5x + 5 = 0$ | | | |

| | | | |
|---|---|---|---|
| $x^2 - 4x + 4 = 0$ | | | |
| $2x^2 + x + 1 = 0$ | | | |

**Use the information above to answer the following questions:**

1. What do you notice about the value of the discriminant for a quadratic equation that has **2 rational roots**?
2. What do you notice about the value of the discriminant for a quadratic equation that has **2 irrational roots**?
3. What do you notice about the value of the discriminant for a quadratic equation that has **1 rational roots**?
4. What do you notice about the value of the discriminant for a quadratic equation that has **2 imaginary roots(zero real roots)**?

## II. Practice: Find the discriminant and describe the nature of the roots.

| | |
|---|---|
| $2x^2 + x - 15 = 0$ | $2x^2 - 9x + 8 = 0$ |
| $\frac{5}{7}x^2 + \frac{4}{7} = \frac{2}{7}x$ | $x^2 = 8x - 16$ |

# Honors Algebra 2: Quadratic Formula and the Discriminant

**Quadratic Formula:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Use standard form of the Quadratic Equation: ______________________.
  1. Get the quadratic in standard form.
  2. Identify a, b, and c.
  3. Substitute into the formula and find the roots.
- Discriminant: the number under the radical in the quadratic formula(___________)
  1. Determines how many real solutions there are to a problem.
  2. If the **discriminant** is **> 0**, there are ________ __________solutions.
  3. If the **discriminant** is **= 0**, there is ________ __________solution.
  4. If the **discriminant** is **< 0**, there are _____real solutions and ___ ____________solutions.

Use the quadratic formula to find the roots.

1) $x^2 - 9x + 5 = 0$

2) $5x^2 + 3x - 1 = 0$

3) $-x^2 + 2x + 4 = 0$

4) $x^2 - 6x + 9 = 0$

5) $x^2 + 4x + 4 = 0$

6) $x^2 + 10x + 25 = 0$

7) $x^2 - 6x = -10$

8) $x^2 = -x - 1$

10) $x^2 - 2x + 3 = 0$

Find the **discriminant** of the quadratic equation and give the number and type of roots.

11) $9x^2 + 6x + 1 = 0$

12) $9x^2 + 6x - 4 = 0$

13) $9x^2 + 6x + 5 = 0$

## Mini-Review

1. Write the expression $(6+3i)+(-4+10i)$ in standard form.

2. Write the expression $(-2+6i)-(2-3i)$ in standard form.

3. Write the expression $(2+5i)(5-i)$ in standard form.

4. Write the expression $\dfrac{2-3i}{6+i}$ in standard form.

5. Solve the equation $x^2-4x+10=0$ using the quadratic formula.

6. Write $y=x^2+8x+1$ in vertex form.

## Day 12: Graphing & Solving Quadratic Inequalities (Text p. 269)

- **Graphing Quadratic Inequalities**
  - Use the same techniques as we used to graph linear inequalities.

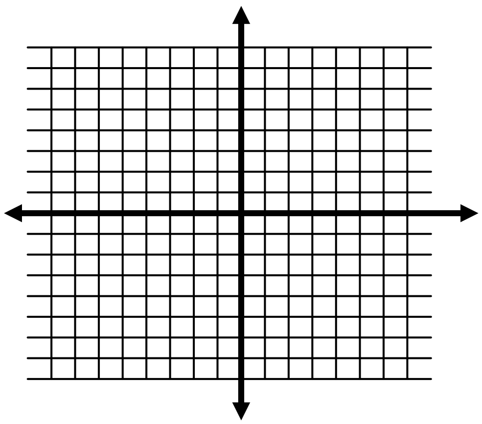
***STEPS***

Graph the boundary. Determine if it should be solid $\leq, \geq$ or dashed (> , <).

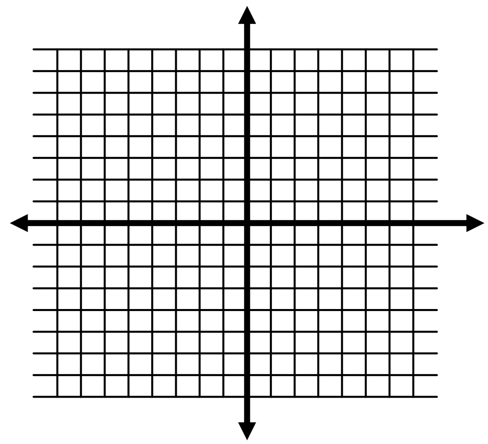
Test a point in each region.

Shade the region whose ordered pair results in a true inequality

EXAMPLE 1: Graph $y \leq x^2-6x+2$



EXAMPLE 2:Graph $y < -x^2+3x-4$



## Solving Quadratic Inequalities

Solve :

1) $0 > x^2 - 6x - 7$ 2) $x^2 + 9x + 14 < 0$ 3) $x^2 - x - 12 \geq 0$

4) $b^2 \geq 10b - 25$ 5) $2x^2 + 5x < 12$ 6) $n^2 \leq 3$

**Interval Notation:**

**For $<$ or $>$ use (**

**For $\leq$ or $\geq$ use [**

**For $\leq$ or $\geq$ use [**

**Practice:**

Solve by graphing: 1. $y \geq 2x^2 + x - 3$ 2. $y \leq -x^2 + 2x + 8$

Solve the inequality algebraically: 3. $x^2 - 3x - 10 < 0$ 4. $x^2 + x \geq 8$

## Solve a linear and Quadratic System by Graphing

Solve:

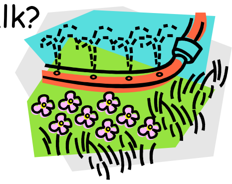
1) $y = x^2 + 5x + 6$
$y = x + 6$

2) $y = -x^2 - x + 12$
$y = x^2 + 7x + 12$

3) $y < -x^2 - 4x + 3$
$y > x^2 + 3$

# Honors Alg 2: Quadratic Applications Day 13

1. A rectangular garden contains 120 $ft^2$ and has a walk of uniform width surrounding it. If the entire area, including the walk is 12ft x 14ft, how wide is the sidewalk?



2. Lorenzo has 48 feet of fencing to make a rectangular dog pen. If a house were used for one side of
 the pen, what would be the length and width for the maximum area?

3. What is the largest area that can be enclosed with 400 feet of fencing? What are the dimensions of the rectangle?

4. The floor of a shed has an area of 54 square feet. Find the length and width if the length is 3 feet less than twice its width.

5. A rectangular pool holds 2400 $ft^2$ of water and is surrounded by a deck. Find the width of this deck if the width is uniform throughout and if the total dimensions are 50ftx70ft.

6. The product of two consecutive negative numbers is 1122. What are the numbers?